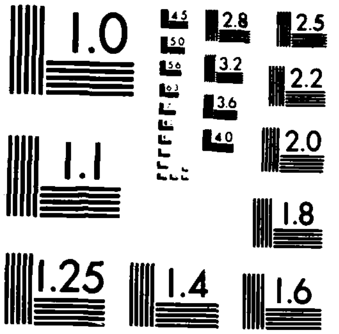
AD-A193 628 EASY BAYES ESTIMATION FOR RASCH-TYPE MODELS(U) SOUTH CAROLINA UNIV COLUMBIA CENTER FOR MACHINE INTELLIGENCE R J JANNARONE ET AL 0 NOV 87 USCMI-87-66 1/1

UNCLASSIFIED N00014-86-K-0817 F/G 12/3 NL



Easy Bayes Estimation for Rasch-Type Models†

Robert J. Jannarone
James E. Laughlin
Kai F. Yu

University of South Carolina

USCMI Report No. 87-66

CENTER FOR MACHINE INTELLIGENCE

UNIVERSITY OF SOUTH CAROLINA

COLUMBIA, SC 29208

This research was sponsored by Personnel and Training Research Programs, Psychological Sciences Division, Office of Naval Research, under Contract No. N00014-86-K00817, Authority Identification Number, NR 4421-544.

4

Easy Bayes Estimation for Rasch-Type Models†

Robert J. Jannarone
James E. Laughlin
Kai F. Yu

University of South Carolina

USCMI Report No. 87-66

4 November, 1987

Key words: Item response theory; conjunctive models, compensatory, reactive measurement, nonadditive measurement, Rasch model.

† Approved for public release; distribution unlimited. Reproduction in whole or in part is permitted for any purpose of the United States Government. This research was sponsored by Personnel and Training Research Programs, Psychological Sciences Division, Office of Naval Research Contract No. N00014-86-K00817, Authority Identification Number, NR 4421-544.

We wish to thank Sheng-Hui Chu and Dzung-Ji Lii for providing intelligent and energetic programming support for this article.

Easy Bayes Estimation for Rasch-Type Models

Robert J. Jannarone, James E. Laughlin and Kai F. Yu

Abstract

A Bayes estimation procedure is introduced that allows the nature and strength of prior beliefs to be easily specified and posterior models to be estimated with no more difficulty than maximum likelihood estimation. The procedure is based on constructing posterior distributions that are formally identical to likelihoods, but are constructed partly from sample data and partly from artificial data reflecting prior information. Improvements in performance of modal Bayes procedures relative to maximum likelihood estimation procedures are illustrated for Rasch-type models. Improvements range from modest to dramatic, depending on the model and the number of items being considered.

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved OMB No 0704-0188

1a REPORT SECURITY CLASSIFICATION	Unclassified
1b RESTRICTIVE MARKINGS	
2a SECURITY CLASSIFICATION AUTHORITY	
2b DECLASSIFICATION / DOWNGRADING SCHEDULE	
3. DISTRIBUTION / AVAILABILITY OF REPORT	Approved for public release; distribution unlimited
4 PERFORMING ORGANIZATION REPORT NUMBER(S)	ONR 86-2
5. MONITORING ORGANIZATION REPORT NUMBER(S)	
6a NAME OF PERFORMING ORGANIZATION	University of South Carolina
6b OFFICE SYMBOL (If applicable)	
6c. ADDRESS (City, State, and ZIP Code)	Columbia, South Carolina 29208
7a. NAME OF MONITORING ORGANIZATION	Office of Naval Research
7b. ADDRESS (City, State, and ZIP Code)	800 N. Quincy St., Code 442 Arlington, VA 22217
8a. NAME OF FUNDING / SPONSORING ORGANIZATION	Personnel & Training Res.
8b. OFFICE SYMBOL (If applicable)	
8c. ADDRESS (City, State, and ZIP Code)	
9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	N00014-86-K-0817

10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.
61153N 42	RR04204	RR0420401	4421-544

11. TITLE (Include Security Classification)

Easy Bayes Estimation for Rasch-Type Models

12. PERSONAL AUTHOR(S)

Robert J. Jannarone, James E. Laughlin and Kai F. Yu

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Technical Report	FROM 8-15-86 TO 11-31-87	11/4/87	12

16 SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

A Bayes estimation procedure is introduced that allows the nature and strength of prior beliefs to be easily specified and posterior models to be estimated with no more difficulty than maximum likelihood estimation. The procedure is based on constructing posterior distributions that are formally identical to likelihoods, but are constructed partly from sample data and partly from artificial data reflecting prior information. Improvements in performance of modal Bayes procedures relative to maximum likelihood likelihood estimation procedures are illustrated for Rasch-type models. Improvements range from modest to dramatic, depending on the model and the number of items being considered.

20 DISTRIBUTION / AVAILABILITY OF ABSTRACT	☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS
21 ABSTRACT SECURITY CLASSIFICATION	unclassified
22a NAME OF RESPONSIBLE INDIVIDUAL	Charles E. Davis
22b TELEPHONE (Include Area Code)	(202) 696-4046
22c OFFICE SYMBOL	

DD Form 1473, JUN 86 Previous editions are obsolete. SECURITY CLASSIFICATION OF THIS PAGE

Unclassified

Easy Bayes Estimation for Rasch-Type Models

Introduction

Scope. Augmenting observed data by artificial observations has been used informally for some time to solve certain estimation problems. For example, adding observations to empty cells in contingency tables was recommended over 35 years ago (Rao, 1952) in order to make joint categorical probabilities estimable. Artificial data augmentation has also been recognized as a useful and general device for incorporating prior beliefs (Jackson & Novick, 1974). Of more direct interest, Wright (1986) recommended adding artificial item scores to individuals' Rasch model test scores, in order to obtain latent trait estimates for individuals who pass all items or fail all items. Although he did not justify the approach formally, Wright also suggested that adding such artificial observations to data corresponds to imposing a kind of Bayes prior. In a recent article, Tanner and Wong (1987) made a more formal connection between artificial data augmentation and Bayes theory. They described a class of corresponding estimation procedures as well. This article describes and justifies a new data augmentation Bayes approach to Rasch-type model estimation that has statistical and computational advantages over existing methods. The Bayes approach may also be used to reflect prior beliefs for Rasch and other exponential family models, in ways that may usefully supplement existing methods.

Existing Bayes methods for Rasch-type models each have their liabilities. Since the Rasch model belongs in the exponential family, conjugate prior and posterior distributions may easily be found (Bickel & Doksum, 1977). However, obtaining satisfactory estimates such as posterior means or posterior modes is often not easy. The same seems true of Bayes and empirical Bayes estimates in test theory (Mislevy, 1986; Tsutakawa & Lin, 1986) as well as those described by Tanner & Wong (1987). Also, although the method described by Wright seems quite simple the method is not justified, especially in terms of a precise Bayes formulation. *Empirical* Bayes approaches have already been suggested that incorporate "auxiliary" information into item response models (Mislevy, 1986; Swaminathan & Gifford, 1981, 1982 and 1985). The Bayes approach described here differs from these methods in three ways. First, in the Bayes procedure we explicitly design our priors to incorporate a *minimal* degree of auxiliary information. In contrast, the amount of prior information that empirical Bayes approaches attribute to the prior is dictated by the data and can be substantial. Second, as with existing Bayes and empirical Bayes approaches we assume exchangeability across relevant model parameters. In contrast, however we explicitly state an *a priori* modal value for the exchangeable parameters in a way that clearly identifies the model. Finally, because we utilize a particular class of conjugate priors we end up with posteriors in the same form as the likelihood. Thus, we easily obtain posterior modal estimates by making minor modifications to existing maximum likelihood (ML) estimation programs.

Purpose. The purpose of this article is to describe and justify a method for easily incorporating prior information through data augmentation, by (a) deriving the method as a posterior modal procedure, given certain conjugate structures; (b) illustrating the method's use for some Rasch-type situations; and (c) demonstrating how the method can be used to considerably improve parameter estimation.

An informal overview and result summary will be given below. Technical details will be described later.

Overview. We will begin by applying the model to the familiar Rasch case, which leads to modest estimation improvements. We will then consider more impressive improvements based on two less familiar models.

When estimating parameters for the Rasch model, problems due to sufficient statistics taking on boundary values can occur if test lengths are small and/or observed score distributions are skewed. In such cases a substantial proportion of individuals may fail all items or pass all items, in which case their latent trait values will not be estimable. Losing such individuals can lead to deflated correlations between estimated latent traits and other variables, because latent trait estimates based on extreme scores will be excluded. In addition, biased estimates of item parameters may result, because the same individual latent trait estimates will not be available for simultaneous item parameter estimation (and consequently estimated latent trait distributions may become distorted). Similar problems may also occur when item parameter sufficient statistics take on boundary values, which can occur occasionally when sample sizes are small.

An easy way to remove such problems is to augment observed data with artificial data such that resulting sufficient statistics cannot take on boundary values. For example, suppose that data were available from a (binary) 6-item test and that scores from two additional items were added to each individual's item score. Suppose further that for each individual exactly one augmented item score was coded "pass" and exactly one was coded "fail". The resulting augmented data would yield test scores from 1 to 7 on an 8-item test instead of scores from 0 to 6 on a 6-item test, with each individual having number-correct scores augmented by 1. Thus, if augmented data were used instead of the raw data for individual parameter estimation, the boundary values would disappear. (Using such an approach to avoid estimation problems of course raises questions including whether or not the procedure is formally justified, how augmented item parameters should be treated, and the extent to which resulting estimates could be distorted. Such questions will be addressed later— for now only the mechanics and global results of the approach will be described.)

The first part of Table 1 indicates the kinds of improvements in correlations between true and estimated latent traits that the above kind of data augmentation can yield. As indicated, all improvements are modest and are evident only in cases involving small numbers of items, M. Also, although reliability improvements (that can be obtained by computing square roots of the Table 1 entries) are greater, they are still modest. In addition, only a small proportion of individuals will be recovered by the data augmentation approach, unless M is small. For example, the proportion of recovered individuals corresponding to I values of 1,000 in Table 1 were .093, .026, and .004 for additive Rasch models based on 6, 10, and 20 items, respectively. Thus, only minor improvements seem likely for the Rasch model, unless M is small and strong floor or ceiling effects are present.

The next example leads to considerably more dramatic improvements, because it yields much more frequently occurring boundary values. In a recent attempt to reflect individual differences in learning abilities, Jannarone (1987) has developed a family of so-called Markov item response models. One of these, called the bivariate Rasch Markov (BRM) model, differs from the usual Rasch model in that two individual parameters are involved instead of only one. One parameter, γ, is analogous to the usual Rasch ability parameter in that its sufficient statistic is the number-correct score for a given individual. The second parameter, δ, reflects individuals' abilities to learn and apply new information to subsequent items. The second parameter's sufficient statistic is the number of times an individual passed item n *as well as* item $n+1$ ($n = 1, \ldots, M-1$).

Figure 1(b) indicates the possible contingencies for individuals' sufficient statistics, given a 10-item test satisfying a BRM model. All possible contingencies lie either on or inside the dark gray perimeter. As indicated, it is never possible for the δ sufficient statistic, d, to be as large as the γ sufficient statistic, g. For example, at most 4 distinct adjacent pairs of items could be passed if only 5 total items were passed. Adjacent cross-product scores also restrict number-correct scores. For example, if only 3 adjacent pairs of items were passed then no more than 8 items in a 10-item test could be passed (otherwise more than 3 pairs would have necessarily been adjacent).

Besides unusual contingency restrictions for the bivariate Rasch Markov case, unusual boundary values occur as well. For example, if g were 8 then the lower and upper boundary values for d would be 5 and 7, respectively. Moreover, such boundary values do not have finite MLE's, just as sufficient statistic values of 0 and M in the Rasch case do not have finite MLE's. Consequently, all such boundary values are inestimable. Similarly, the smallest and largest g values for fixed d values are also inestimable. All such inestimable cells for the 10-item case are indicated by dark gray squares in Figure 1(b). Likewise, all inestimable cells for the 17-item case are indicated by light gray squares in Figure 1(a).

As Figures 1(a) and (b) indicate, many cells are inestimable for BRM cases—many more than for the Rasch case. Consequently, much larger proportions of individuals must be excluded than in the Rasch case. For example, in the Table 1 bivariate Markov simulations with I values of 1,000 and M values of 6, 10, and 20, the proportions of randomly generated individuals that were excluded from ML estimation were .949, .745, and .354, respectively.

The boundary problem can be solved for the BRM case in much the same way as in the Rasch case-by augmenting individuals' observed test scores with artificial item scores. In the BRM case, the minimal raw score augmenting solution entails adding 7 items to all individuals' test patterns, such that each g value becomes augmented by 3 and each d value becomes augmented by 1. The consequence of one such augmentation is illustrated in Figure 1(c) for the 10-item case. As indicated all of the original 10-item contingencies will occur within the 17-item boundary values, once they have been augmented by artificial data in this way.

Table 1*

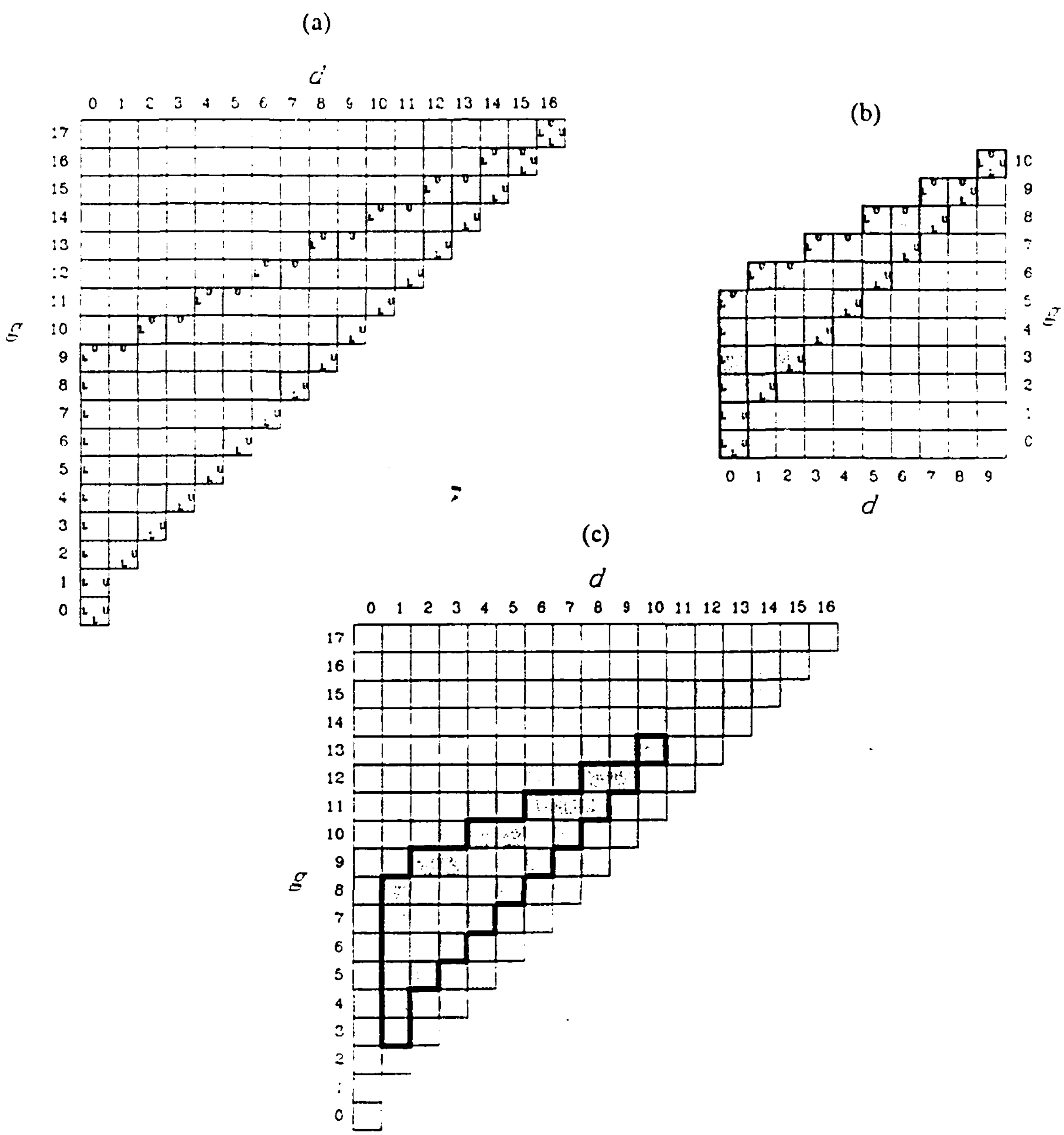
True-Estimated Individual Parameter Correlations
For Maximum Likelihood and Bayes Estimates.*

Model	Number of Items (M)	Sample Size (I)		True Score/Maximum-Likelihood-Estimate Correlation	True Score/Bayes-Estimate Correlation
Additive Rasch	6	100		.63	.66
	6	1000		.68	.72
	10	100		.75	.86
	10	1000		.79	.82
	20	100		.87	.86
	20	1000		.88	.89
	30	100		.92	.91
	30	1000		.92	.92
Bivariate Rasch Markov	6	100	γ	-	.51
			δ	-	.36
	6	1000	γ	-	.55
			δ	-	.28
	10	100	γ	.16	.68
			δ	.20	.26
	10	1000	γ	.30	.60
			δ	.33	.38
	10	5000	γ	.33	.61
			δ	.38	.43
	15	100	γ	.23	.63
			δ	.40	.54
	15	1000	γ	.51	.62
			δ	.54	.51
	20	100	γ	.55	.67
			δ	.53	.55
	20	1000	γ	.59	.67
			δ	.60	.57

*Entries are product-moment sample correlations. For additive cases latent trait values were randomly sampled from 5 points, -2, -1, 0, 1, and 2, having (quasinormal) probabilities, .07, .24, .38, .24, and .07, respectively. For bivariate cases latent trait values were randomly sampled from 25 points, (-2,-2), (-2,-1), . . . , (2,2) such that marginal probabilities were the same as in the additive case and the two latent traits were mutually independent. For additive Rasch models half of the item difficulties were +1 and half were -0.5. For bivariate Rasch Markov models the additive item parameters were 1.0 and the cross-product item parameters were -0.5. estimates were obtained by a Newton-Raphson approach described in the text and in Jannarone (1987). For both models all (Bayes *vs.* nonBayes) random samples were obtained independently.

Figure 1.

Individual Sufficient Statistic Features for Bivariate Rasch Markov Tests of Length 10 and 17.*



* Figures 1(a) and (b) correspond to $M = 17$ and $M = 10$, respectively. For both Figures the possible (g, d) contingencies include boundary values that are shaded as well as estimable contingencies that are unmarked and inside the boundary-value perimeter. Contingencies corresponding to lower g (d) bounds are labelled by L's at the bottom (left side) of shaded squares, whereas contingencies corresponding to upper g (d) values are labelled by U's at the top (right side) of shaded squares. Figure 1 (c) illustrates how the 10-item contingencies would all lie within the 17-item boundary perimeter, if d, g, and M for the 10-item case were transformed to $d + 1$, $g + 3$ and $M + 7$, respectively.

The entries in the bottom of Table 1 indicate the dramatic improvements in validity that can be expected from artificial data augmentation for the BRM case. For the 6-item case it is not even possible to correlate individual parameter MLE's with other variables because only one cell is estimable. For other cases, improvements in both γ and δ estimates are strong, even for moderate M values.

Besides solving boundary value problems, artificial data augmentation can be easily used to impose prior structures on data (Novick & Jackson. 1974). For example, Jannarone, Yu, and Takefuji (1987) have recently developed a set of conjunctive models for neural and machine learning. One purpose of such models is to accurately estimate associations between one (input) binary vector and another (output) binary vector over a series of learning trials. In each learning trial, a datum consisting of joint (input, output) values for the vectors is presented and the model must specify how much weight to give the learning trial datum, relative to the previous learning trial data and/or "prior beliefs". A detailed description of the mechanism for incorporating such learning trial weighting is beyond this article's scope. We merely mention that the mechanism corresponds precisely to augmenting each learning trial datum with "prior" artificial data. The data augmentation mechanism for that case is also quite easy to implement and interpret. One of the simpler models that could be used this way, called the Rasch Markov model with no individual differences, will be described in the next section.

Regarding distortions that could arise from artificial data augmentation, the augmentation process corresponds formally to a Bayes posterior estimation scheme, as will be shown below. Consequently, the process can lead to biased estimates just as any Bayes procedure can lead to biased estimates. However, as for many other Bayes procedures the bias will not be serious in that (a) bias in the cases that we consider here corresponds to a uniform shrinkage of parameter estimates toward some central value; (b) the Bayes estimates that result from the augmentation process will always be monotonically related to maximum likelihood estimates; and (c) bias levels will decrease as the sample sizes and/or numbers of items increase. Moreover, in some cases incorporating bias through such data augmentation may actually be helpful toward adjusting item parameter estimates that are known to biased. One such application might be in estimating item parameters, for example see (Samejima, 1987).

In the next section we will connect artificial data augmentation with Bayesian prior/posterior probability structures. Besides pointing toward appropriate estimation schemes and proper interpretations, the results to follow will also suggest ways that data augmentation can lead to model identification.

Detailed Description

Conjugate cases for exponential families. Although the following approach seems to have general utility, only observables having binary elements will be considered here. For any sample consisting of I M-variate observations, $\mathbf{x}_1, \ldots, \mathbf{x}_I$, and having a likelihood of the natural exponential family form,

$$L(\mathbf{x}_1, \ldots, \mathbf{x}_I \mid \underset{1\times M}{\boldsymbol{\alpha}}) = [\nu(\boldsymbol{\alpha})]^I \exp\{\sum_{r=1}^{R} \alpha_r \sum_{i=1}^{I} s_r(\mathbf{x}_i)\},\ \mathbf{x}_i \in \tilde{B}^M,\ i = 1, \ldots, I, \tag{1}$$

where the

$$\sum_{i=1}^{I} s_r(\mathbf{x}_i),\quad r = 1, \ldots, R$$

are sufficient statistics corresponding to the parameters α_1, through α_R,

$$\nu(\boldsymbol{\alpha}) = \left[\sum_{\mathbf{u} \in B^M} \exp\{\sum_{r=1}^{R} \alpha_r s_r(\mathbf{u})\}\right]^{-1},$$

and

$$\tilde{B}^M = \{\underset{M\times 1}{\mathbf{u}} : u_m = 0,1,\ \ m = 1, \ldots, M\}:$$

a (possibly improper) conjugate prior density is given by

$$f(\boldsymbol{\alpha} \mid \mathbf{A}, J) \propto \nu(\boldsymbol{\alpha})^J \exp\{\sum_{r=1}^{R} \alpha_r A_r\} \tag{2}$$

(Bickel & Doksum, 1977, Prop. 2.3.1—the conjugate prior will be proper for a given $\mathbf{A}$ and J if

$$\int_{\alpha \in \tilde{R}^R} [\nu(\alpha)]^J \exp\{ \sum_{r=1}^{R} \alpha_r A_r \} d\alpha < \infty. \ )$$

A consequence of (1) and (2) is that the posterior probability function,

$$h(\alpha \mid \mathbf{x}_1, \ldots, \mathbf{x}_I, \mathbf{A}, J) = L(\mathbf{x}_1, \ldots, \mathbf{x}_I \mid \alpha) f(\alpha \mid \underset{1 \times R}{\mathbf{A}}, J)$$

$$\propto \nu(\alpha)^{I+J} \exp\{ \sum_{r=1}^{R} \alpha_r (\sum_{i=1}^{I} s_r(\mathbf{x}_i) + A_r) \} , \tag{3}$$

has the same parametric from as (1), that is, (3) is conjugate to (1). For example, if the $\mathbf{x}_i$ satisfy a Rasch Markov model (Jannarone, 1987) with no individual differences, then

$$L(\mathbf{x}_1, \ldots, \mathbf{x}_I \mid \underset{1 \times (2M-1)}{\beta}) = [\nu(\beta)]^I \exp\left\{ \sum_{m=1}^{M} \beta_m \sum_{i=1}^{I} x_{im} + \sum_{n=1}^{M-1} \beta_{n,n+1} \sum_{i=1}^{I} x_{in} x_{i,in+1} \right\},$$

so that a conjugate prior density is given by,

$$f(\beta \mid \underset{1 \times (2M-1)}{\mathbf{B}}, J) \propto [\nu(\beta)]^J \exp\left\{ \sum_{m=1}^{M} \beta_m B_m + \sum_{n=1}^{M-1} \beta_{n,n+1} B_{n,n+1} \right\},$$

which leads to the posterior probability function,

$$h(\beta \mid \mathbf{x}_1, \ldots, \mathbf{x}_I, \mathbf{B}, J) \propto [\nu(\beta)]^{I+J} \exp\left\{ \sum_{m=1}^{M} \beta_m (\sum_{i=1}^{I} x_{im} + B_m) + \sum_{n=1}^{M-1} \beta_{n,n+1} (\sum_{i=1}^{I} x_{in} x_{i,n+1} + B_{n,n+1}) \right\}.$$

Conjugating prior densities. Just as conjugate prior densities have the same parametric form as their resulting posteriors, priors may be constructed such that their likelihoods and posterior probability functions have the same parametric form. Such priors will be called ***conjugating*** because they impose conjugacy between posteriors and likelihoods rather than between posteriors and themselves. Conjugating cases are particularly interesting when resulting posterior probability functions correspond to likelihoods for feasible i.i.d. samples. In the sequel we will restrict the meaning of conjugating to include only priors that yield such feasible "posterior likelihoods".

The structure of (1), (2), and (3) suggests a simple method for obtaining conjugating priors for exponential family likelihoods. For a given likelihood and prior satisfying (1) and (2), the resulting posterior (3) will be a feasible likelihood from the same family as (1) if $I+J$ is a positive integer and the

$$\sum_{i=1}^{I} s_r(\mathbf{x}_i) + A_r$$

are feasible sufficient statistics from a sample of size $I+J$. That is, for a likelihood of form (2) a conjugate prior of form (1) will also be conjugating if there exist $z_1, \ldots, z_J \in B^M$ such that

$$A_r = \sum_{j=1}^{J} s_r(z_j), \quad r = 1, \ldots, R.$$

(Similar methods have been suggested previously for other applications— see Novick & Jackson 1974.)

One useful feature of conjugating priors is the ease with which they can reflect prior information. Conjugating priors can be imposed such that the strength of prior belief is indicated by prior sample sizes and the nature of prior belief is indicated by prior sufficient statistic values. Returning to the Rasch Markov example with no individual differences, suppose that one wished to combine data with the prior notion that the elements in $\mathbf{X}$ were mutually independent and identically Bernoulli (0.5). The relative degree of prior belief would be indicated by the size of J relative to I— for instance equal prior and data weightings would correspond to $I = J$. The nature of prior beliefs in this case would correspond to setting $\mathbf{B} = \mathbf{0}$. (This and similar cases have been extended in neural and machine learning settings to include noninteger values for J within the context of "learning trial weightings"— see Jannarone, Yu, & Takefuji, 1987 for details.)

A second feature of conjugating priors is the ease with which they can yield posterior estimates. First, for models satisfying (1) unique MLE's exist whenever sufficient statistics are not boundary values. Second, provisions for obtaining MLE's are available in many such cases (including the Rasch Markov case—Jannarone, 1987). As a consequence of the conjugating property such procedures may be used to find posterior modes, because posterior modes are formally equivalent to likelihood maxima, given the conjugating property.

A third conjugating prior feature, which motivated this article, is the potential for solving problems due to boundary-valued sufficient statistics. As a first example consider Rasch model estimation based on the likelihood,

$$L(\mathbf{x}_1, \ldots, \mathbf{x}_I \mid \underset{1\times I}{\boldsymbol{\theta}}, \underset{1\times M}{\boldsymbol{\beta}}) = \left[\prod_{i=1}^{I}\{\prod_{m=1}^{M}(1+\exp\{\theta_i-\beta_m\})\}\right]^{-1} \exp\left\{\sum_{i=1}^{I}\sum_{m=1}^{M}(\theta_i-\beta_m)x_{im}\right\}$$

$$= c(\boldsymbol{\theta}, \boldsymbol{\beta})\exp\left\{\sum_{i=1}^{I}\theta_i\sum_{m=1}^{M}x_{im} - \sum_{m=1}^{M}\beta_m\sum_{i=1}^{I}x_{im}\right\},$$

where $\boldsymbol{\theta}$ and $\boldsymbol{\beta}$ contain individual and person parameters, respectively. Parameter estimation problems arise in the Rasch case when sufficient statistics take on their minimum or maximum possible values. Besides leading to inestimable individual parameters the problem can also lead to biased item parameter estimates, because item parameter MLE's depend on individual parameter MLE's.

A conjugating prior for solving Rasch model boundary problems can be constructed as follows. (The following process for constructing conjugating priors differs slightly from the conjugate-prior-based example given previously—although the process yields posteriors that are also formally equivalent to Rasch likelihoods, the resulting posteriors will be based on different numbers of items than their corresponding likelihoods.) By setting

$$f(\boldsymbol{\theta}, \boldsymbol{\beta}) \propto \left[\prod_{i=1}^{I}\prod_{n=1}^{2}(1+\exp\{\theta_i\})\right]^{-1} \exp\{\sum_{i=1}^{I}(\theta_i)\},$$

the "posterior likelihood" takes the form,

$$\left[\prod_{i=1}^{I}\prod_{m=1}^{M}(1+\exp\{\theta_i-\beta_m\})\right]^{-1}\left[\prod_{i=1}^{I}\prod_{n=1}^{2}(1+\exp\{\theta_i-0\})\right]^{-1} \times$$

$$\exp\left\{\sum_{i=1}^{I}\left[\sum_{m=1}^{M}(\theta_i-\beta_m)x_{im}+(\theta_i-0)1+(\theta_i-0)\,0\right]\right\}. \qquad (4)$$

The posterior (4) is clearly equivalent to a likelihood from an $(M+2)$-item test, with each individual's observed M-item score augmented by a score of 1 on a subtest based on two additional items, each having a difficulty of 0. Thus, the prior information for $\boldsymbol{\theta}$ is exchangeable and reflects an *a priori* modal estimate of zero. Also, the weight associated with this prior information can be represented by the ratio of hypothetical to actual test items, in this case, $2/M$. The prior weight is minimal in that two hypothetical items are necessary to resolve the boundary value problem in the Rasch model.

Interestingly, the difficulty scale becomes implicitly identified by the prior (4) in that a difficulty value of zero is associated with the two artificial items. (In the empirical Bayes procedures cited previously the data determine, in an uncertain way, the identification of the difficulty scale, whereas the usual Rasch model requires fixing one parameter during estimation for identifiability.)

The gradient elements for the logarithm of the posterior (4) take the form,

$$\frac{\partial \mathrm{L}}{\partial \beta_m} = -\sum_{m=1}^{M} x_{im} + \sum_{i=1}^{I}\frac{\exp\{\theta_i-\beta_m\}}{1+\exp\{\theta_i-\beta_m\}}, \quad m = 1, \ldots, M. \qquad (5)$$

and

$$\frac{\partial \mathrm{L}}{\partial \theta_i} = \sum_{m=1}^{M} x_{im}+1 - \left[\sum_{m=1}^{M}\frac{\exp\{\theta_i-\beta_m\}}{1+\exp\{\theta_i-\beta_m\}} + \frac{2\exp\{\theta_i\}}{1+\exp\{\theta_i\}}\right], \quad i = 1, \ldots, I. \qquad (6)$$

The posterior modal estimate (PME) $\boldsymbol{\beta}$ gradients in (5) are identical to the usual Rasch model log-likelihood gradients (Andersen, 1980). Also, the PME $\boldsymbol{\theta}$ gradients in (6) are identical to MLE $\boldsymbol{\theta}$ gradients, except individual sufficient statistics are augmented by 1 and two additional item parameters are involved, each having 0-valued parameters. Thus, Rasch PME's may be obtained by making only minor modifications to existing Rasch MLE procedures.

The remaining PME example, which was illustrated earlier in Figure 1, imposes conjugating prior structure on bivariate Rasch Markov person parameters and results in major estimation improvements. For this case likelihoods take the form (Jannarone, 1987),

[illegible]

$$\exp\left\{\sum_{i=1}^{I}\left[\sum_{m=1}^{M}(\gamma_i-\beta_m)x_{im}+\sum_{n=1}^{M-1}(\delta_i-\beta_{n,n+1})x_{in}x_{i,n+1}\right]\right\};$$

(minimally informative boundary-value removing) conjugating priors take the form,

$$f(\gamma,\ \delta,\ \beta)\propto\left[\prod_{i=1}^{I}\ \sum_{v\in B^7}\exp\{\sum_{m=1}^{7}\gamma_i v_m+\sum_{n=1}^{6}\delta_i v_n v_{n+1}\}\right]^{-1}\exp\{\sum_{i=1}^{I}(3\gamma_i+\delta_i)\}\ ;$$

and resulting posteriors are,

$$\begin{aligned}h(\gamma,\ \delta\mid L,\ F)\propto&\left[\prod_{i=1}^{I}\ \sum_{u\in B^M}\exp\{\sum_{m=1}^{M}(\gamma_i-\beta_m)u_m+\sum_{n=1}^{M-1}(\delta_i-\beta_{n,n+1})u_n u_{n+1}\right]^{-1}\times\\&\left[\prod_{i=1}^{I}\ \sum_{v\in B^7}\exp\{\sum_{m=1}^{7}\gamma_i v_m+\sum_{n=1}^{6}\delta_i v_{n_{n+1}}\}\right]^{-1}\times\\&\left[\exp\left\{\sum_{i=1}^{I}\ \sum_{m=1}^{M}(\gamma_i-\beta_m)x_{im}+\sum_{m=1}^{3}(\gamma_i-0)1+\sum_{m=4}^{7}(\gamma_i-0)\,0\,+\right.\right.\\&\left.\left.\sum_{n=1}^{M-1}(\delta_i-\beta_{n,n+1})x_{in}x_{i,n+1}+(\delta_i-0)1\ +\ \sum_{n=2}^{6}(\delta_i-0)\,0\right\}\right]^{-1}.\end{aligned}$$

As in the additive Rasch case, PME item parameter gradients are the same as their MLE counterparts (given in Jannarone, 1987), whereas individual parameters may be estimated by simply augmenting MLE sufficient statistics and including a small number of additional 0-valued item parameters.

Summary

An easy method for incorporating prior Bayes information into Rasch-type model estimation has been described in this article. The method focuses on constructing prior probabilities so that including prior information is equivalent to augmenting sample data with artificial data. Consequently, (a) such prior probability structures conjugate likelihoods with resulting posterior distributions; (b) the nature of prior belief is reflected by "prior sufficient statistic values"; (c) the degree of prior belief is reflected by "prior sample sizes"; and (d) posterior modal estimation entails no more difficulty than maximum likelihood estimation. In addition, empirical results based on simulated data have been provided, showing that the method removes boundary valued sufficient statistics for some models. The simulated results indicate modest improvements in Rasch model estimation performance, but dramatic improvements in Rasch Markov estimation performance.

References

Andersen, E. B. (1980). *Discrete Statistical Models with Social Science Applications.* Amsterdam: North Holland.

Bickel, P. & Doksum, K. (1977). *Mathematical Statistics: Basic Ideas and Selected Topics.* San Francisco: Holden-Day.

Jannarone, R. J. (1987). Locally dependent models for reflecting learning abilities. *Psychometrika* (in review).

Jannarone, R. J., Yu, Kai, F. & Takefuji, Y. (1987). "Conjunctoids": Probabilitic learning models for binary events. Unpublished manuscript.

Mislevy, R. J. (1986). Bayes modal estimation in item response models. *Psychometrika, 51,* 171-195.

Novick, M. R. & Jackson, P. H. (1974). *Statistical Methods for Educational and Psychological Research.* New York: McGraw-Hill.

Samejima, F. (1987). Bias Function of the Maximum Likehood Estimate of Ability for Discrete Item Response. Report # ONR/RR 87-1. University of Tennessee.

Swaminathan, H. & Gifford, J. A. (1981). Bayesian estimation for the three-parameter logistic model. Paper presented at the annual Psychometric Society meetings, Chapel Hill, N.C.

Swaminathan, H. & Gifford, J. A. (1982). Bayesian estimation in the Rasch model. *Journal of Educational Statistics, 7,* 175-197.

Swaminathan, H. & Gifford, J. A. (1985). Bayesian estimation for the one-parameter logistic model. *Psychometrika, 47,* 397-412.

Tanner, M & Wong, W. H. (1987). The Calculation of Posterior Distributions by Data Augmentation. *Journal of the American Statistical Association, 82,* 528-540.

Tsutakawa, R. K., & Lin, H. Y. (1986). Bayesian estimation of item response curves. *Psychometrika, 51,* 251-267.

Wright B. (1986). Bayes' answer to perfection. Unpublished Manuscript.

Dr. Michael Levine
Educational Psychology
210 Education Bldg.
University of Illinois
Champaign, IL 61801

Dr. Charles Lewis
Educational Testing Service
Princeton, NJ 08541

Dr. Robert Linn
College of Education
University of Illinois
Urbana, IL 61801

Dr. Robert Lockman
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. Frederic M. Lord
Educational Testing Service
Princeton, NJ 08541

Dr. George B. Macready
Department of Measurement
Statistics & Evaluation
College of Education
University of Maryland
College Park, MD 20742

Dr. Milton Maier
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. William L. Maloy
Chief of Naval Education
and Training
Naval Air Station
Pensacola, FL 32508

Dr. Gary Marco
Stop 31-E
Educational Testing Service
Princeton, NJ 08451

Dr. Clessen Martin
Army Research Institute
5001 Eisenhower Blvd.

Dr. James McBride
Psychological Corporation
c/o Harcourt, Brace,
Javanovich Inc.
1250 West 6th Street
San Diego, CA 92101

Dr. Clarence McCormick
HQ, MEPCOM
MEPCT-P
2500 Green Bay Road
North Chicago, IL 60064

Dr. Robert McKinley
Educational Testing Service
20-P
Princeton, NJ 08541

Dr. James McMichael
Technical Director
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Barbara Means
Human Resources
Research Organization
1100 South Washington
Alexandria, VA 22314

Dr. Robert Mislevy
Educational Testing Service
Princeton, NJ 08541

Dr. William Montague
NPRDC Code 13
San Diego, CA 92152-6800

Ms. Kathleen Moreno
Navy Personnel R&D Center
Code 62
San Diego, CA 92152-6800

Headquarters, Marine Corps
Code MPI-20
Washington, DC 20380

Dr. W. Alan Nicewander
University of Oklahoma
Department of Psychology
Oklahoma City, OK 73069

Deputy Technical Director
NPRDC Code 01A
San Diego, CA 92152-6800

Director, Training Laboratory,
NPRDC (Code 05)
San Diego, CA 92152-6800

Director, Manpower and Personnel
Laboratory,
NPRDC (Code 06)
San Diego, CA 92152-6800

Director, Human Factors
& Organizational Systems Lab,
NPRDC (Code 07)
San Diego, CA 92152-6800

Fleet Support Office,
NPRDC (Code 301)
San Diego, CA 92152-6800

Library, NPRDC
Code P201L
San Diego, CA 92152-6800

Commanding Officer,
Naval Research Laboratory
Code 2627
Washington, DC 20390

Dr. Harold F. O'Neil, Jr.
School of Education - WPH 801
Department of Educational
Psychology & Technology
University of Southern California
Los Angeles, CA 90089-0031

Dr. James Olson
WICAT, Inc.
1875 South State Street
Orem, UT 84057

Office of Naval Research,
Code 1142CS
800 N. Quincy Street
Arlington, VA 22217-5000
(6 Copies)

Office of Naval Research,
Code 125
800 N. Quincy Street
Arlington, VA 22217-5000

Assistant for MPT Research,
Development and Studies
OP 01B7
Washington, DC 20370

Dr. Judith Orasanu
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. Jesse Orlansky
Institute for Defense Analyses
1801 N. Beauregard St.
Alexandria, VA 22311

Dr. Randolph Park
Army Research Institute
5001 Eisenhower Blvd.
Alexandria, VA 22333

Wayne M. Patience
American Council on Education
GED Testing Service, Suite 20
One Dupont Circle, NW
Washington, DC 20036

Dr. James Paulson
Department of Psychology
Portland State University
P.O. Box 751
Portland, OR 97207

Administrative Sciences Department,
Naval Postgraduate School
Monterey, CA 93940

Department of Operations Research,
Naval Postgraduate School
Monterey, CA 93940

Dr. Mark D. Reckase
ACT
P. O. Box 168
Iowa City, IA 52243

Dr. Malcolm Ree
AFHRL/MP
Brooks AFB, TX 78235

Dr. Barry Riegelhaupt
HumRRO
1100 South Washington Street
Alexandria, VA 22314

Dr. Carl Ross
CNET-PDCD
Building 90
Great Lakes NTC, IL 60088

Dr. J. Ryan
Department of Education
University of South Carolina
Columbia, SC 29208

Dr. Fumiko Samejima
Department of Psychology
University of Tennessee
310B Austin Peay Bldg.
Knoxville, TN 37916-0900

Mr. Drew Sands
NPRDC Code 62
San Diego, CA 92152-6800

Lowell Schoer
Psychological & Quantitative
Foundations
College of Education
University of Iowa
Iowa City, IA 52242

Dr. Mary Schratz
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Dan Segall
Navy Personnel R&D Center
San Diego, CA 92152

Dr. W. Steve Sellman
OASD(MRA&L)
2B269 The Pentagon
Washington, DC 20301

Dr. Kazuo Shigemasu
7-9-24 Kugenuma-Kaigan
Fujusawa 251
JAPAN

Dr. William Sims
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. H. Wallace Sinaiko
Manpower Research
and Advisory Services
Smithsonian Institution
801 North Pitt Street
Alexandria, VA 22314

Dr. Richard E. Snow
Department of Psychology
Stanford University
Stanford, CA 94306

Dr. Richard Sorensen
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Paul Speckman
University of Missouri
Department of Statistics
Columbia, MO 65201

Dr. Judy Spray
ACT
P.O. Box 168
Iowa City, IA 52243

Dr. Martha Stocking
Educational Testing Service
Princeton, NJ 08541

Dr. Peter Stoloff
Center for Naval Analysis
200 North Beauregard Street
Alexandria, VA 22311

Dr. William Stout
University of Illinois
Department of Statistics
101 Illini Hall
725 South Wright St.
Champaign, IL 61820

Dr. Hariharan Swaminathan
Laboratory of Psychometric and
Evaluation Research
School of Education
University of Massachusetts
Amherst, MA 01003

Mr. Brad Sympson
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. John Tangney
AFOSR/NL
Bolling AFB, DC 20332

Dr. Kikumi Tatsuoka
CERL
252 Engineering Research
Laboratory
Urbana, IL 61801

Dr. Maurice Tatsuoka
220 Education Bldg
1310 S. Sixth St.
Champaign, IL 61820

Dr. David Thissen
Department of Psychology
University of Kansas
Lawrence, KS 66044

Mr. Gary Thomasson
University of Illinois
Educational Psychology
Champaign, IL 61820

Dr. Robert Tsutakawa
University of Missouri
Department of Statistics
222 Math. Sciences Bldg.
Columbia, MO 65211

Dr. Ledyard Tucker
University of Illinois
Department of Psychology
603 E. Daniel Street
Champaign, IL 61820

Dr. Vern W. Urry
Personnel R&D Center
Office of Personnel Management
1900 E. Street, NW
Washington, DC 20415

Dr. David Vale
Assessment Systems Corp.
2233 University Avenue
Suite 310
St. Paul, MN 55114

Dr. Frank Vicino
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Howard Wainer
Division of Psycholo
Educational Testing
Princeton, NJ 08541

Dr. Ming-Mei Wang
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. Thomas A. Warm
Coast Guard Institut
P. O. Substation 18
Oklahoma City, OK 7

Dr. Brian Waters
Program Manager
Manpower Analysis P
HumRRO
1100 S. Washington
Alexandria, VA 2231

Dr. David J. Weiss
N660 Elliott Hall
University of Minne
75 E. River Road
Minneapolis, MN 554

Dr. Ronald A. Weitz
NPS, Code 54Wz
Monterey, CA 92152-

Major John Welsh
AFHRL/MOAN
Brooks AFB, TX 782

Dr. Douglas Wetzel
Code 12
Navy Personnel R&D
San Diego, CA 9215

Dr. Rand R. Wilcox
University of Sout
California
Department of Psyc
Los Angeles, CA 90

German Military Representative
ATTN: Wolfgang Wildegrube
Streitkraefteamt
D-5300 Bonn 2
4000 Brandywine Street, NW
Washington, DC 20016

Dr. Bruce Williams
Department of Educational
Psychology
University of Illinois
Urbana, IL 61801

Dr. Hilda Wing
NRC GF-176
2101 Constitution Ave
Washington, DC 20418

Dr. Martin F. Wiskoff
Navy Personnel R & D Center
San Diego, CA 92152-6800

Mr. John H. Wolfe
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. George Wong
Biostatistics Laboratory
Memorial Sloan-Kettering
Cancer Center
1275 York Avenue
New York, NY 10021

Dr. Wallace Wulfeck, III
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Kentaro Yamamoto
Educational Testing Service
Rosedale Road
Princeton, NJ 08541

Dr. Wendy Yen
CTB/McGraw Hill
Del Monte Research Park
Monterey, CA 93940

Dr. Joseph L. Young
Memory & Cognitive
Processes
National Science Foundation
Washington, DC 20550

Dr. Anthony R. Zara
National Council of State
Boards of Nursing, Inc.
625 North Michigan Ave.
Suite 1544
Chicago, IL 60611

Dr. Terry Ackerman
American College Testing Programs
P.O. Box 168
Iowa City, IA 52243

Dr. Robert Ahlers
Code N711
Human Factors Laboratory
Naval Training Systems Center
Orlando, FL 32813

Dr. James Algina
University of Florida
Gainesville, FL 32605

Dr. Erling B. Andersen
Department of Statistics
Studiestraede 6
1455 Copenhagen
DENMARK

Dr. Eva L. Baker
UCLA Center for the Study
of Evaluation
145 Moore Hall
University of California
Los Angeles, CA 90024

Dr. Isaac Bejar
Educational Testing Service
Princeton, NJ 08450

Dr. Menucha Birenbaum
School of Education
Tel Aviv University
Tel Aviv, Ramat Aviv 69978
ISRAEL

Dr. Arthur S. Blaiwes
Code N711
Naval Training Systems Center
Orlando, FL 32813

Dr. Bruce Bloxom
Defense Manpower Data Center
550 Camino El Estero,
Suite 200
Monterey, CA 93943-3231

Dr. R. Darrell Bock
University of Chicago
NORC
6030 South Ellis
Chicago, IL 60637

Cdt. Arnold Bohrer
Sectie Psychologisch Onderzoek
Rekruterings-En Selectiecentrum
Kwartier Koningen Astrid
Bruijnstraat
1120 Brussels, BELGIUM

Dr. Robert Breaux
Code N-095R
Naval Training Systems Center
Orlando, FL 32813

Dr. Robert Brennan
American College Testing
Programs
P. O. Box 168
Iowa City, IA 52243

Dr. Lyle D. Broemeling
ONR Code 1111SP
800 North Quincy Street
Arlington, VA 22217

Mr. James W. Carey
Commandant (G-PTE)
U.S. Coast Guard
2100 Second Street, S.W.
Washington, DC 20593

Dr. James Carlson
American College Testing
Program
P.O. Box 168
Iowa City, IA 52243

Dr. John B. Carroll
409 Elliott Rd.
Chapel Hill, NC 27514

Dr. Robert Carroll
OP 0187
Washington, DC 20370

Mr. Raymond E. Christal
AFHRL/MOE
Brooks AFB, TX 78235

Dr. Norman Cliff
Department of Psychology
Univ. of So. California
University Park
Los Angeles, CA 90007

Director,
Manpower Support and
Readiness Program
Center for Naval Analysis
2000 North Beauregard Street
Alexandria, VA 22311

Dr. Stanley Collyer
Office of Naval Technology
Code 222
800 N. Quincy Street
Arlington, VA 22217-5000

Dr. Hans Crombag
University of Leyden
Education Research Center
Boerhaavelaan 2
2334 EN Leyden
The NETHERLANDS

Dr. Timothy Davey
Educational Testing Service
Princeton, NJ 08541

Dr. C. M. Dayton
Department of Measurement
Statistics & Evaluation
College of Education
University of Maryland
College Park, MD 20742

Dr. Ralph J. DeAyala
Measurement, Statistics,
and Evaluation
Benjamin Building
University of Maryland
College Park, MD 20742

Dr. Dattprasad Divgi
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. Hei-Ki Dong
Bell Communications Research
6 Corporate Place
PYA-1k226
Piscataway, NJ 08854

Dr. Fritz Drasgow
University of Illinois
Department of Psychology
603 E. Daniel St.
Champaign, IL 61820

Defense Technical
Information Center
Cameron Station, Bldg 5
Alexandria, VA 22314
Attn: TC
(12 Copies)

Dr. Stephen Dunbar
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. James A. Earles
Air Force Human Resources Lab
Brooks AFB, TX 78235

Dr. Kent Eaton
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. John M. Eddins
University of Illinois
252 Engineering Research
Laboratory
103 South Mathews Street
Urbana, IL 61801

Dr. Susan Embretson
University of Kansas
Psychology Department
426 Fraser
Lawrence, KS 66045

Dr. George Englehard, Jr.
Division of Educational Studies
Emory University
201 Fishburne Bldg.
Atlanta, GA 30322

Dr. Benjamin A. Fairbank
Performance Metrics, Inc.
5825 Callaghan
Suite 225
San Antonio, TX 78228

Dr. Pat Federico
Code 511
NPRDC
San Diego, CA 92152-6800

Dr. Leonard Feldt
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. Richard L. Ferguson
American College Testing
Program
P.O. Box 168
Iowa City, IA 52240

Dr. Gerhard Fischer
Liebiggasse 5/3
A 1010 Vienna
AUSTRIA

Dr. Myron Fischl
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Prof. Donald Fitzgerald
University of New England
Department of Psychology
Armidale, New South Wales 2351
AUSTRALIA

Mr. Paul Foley
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Alfred R. Fregly
AFOSR/NL
Bolling AFB, DC 20332

Dr. Robert D. Gibbons
Illinois State Psychiatric Inst.
Rm 529W
1601 W. Taylor Street
Chicago, IL 60612

Dr. Janice Gifford
University of Massachusetts
School of Education
Amherst, MA 01003

Dr. Robert Glaser
Learning Research
& Development Center
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 15260

Dr. Bert Green
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

Dipl. Pad. Michael W. Habon
Universitat Dusseldorf
Erziehungswissenschaftliches
Universitatsstr. 1
D-4000 Dusseldorf 1
WEST GERMANY

Dr. Ronald K. Hambleton
Prof. of Education & Psychology
University of Massachusetts
at Amherst
Hills House
Amherst, MA 01003

Dr. Delwyn Harnisch
University of Illinois
51 Gerty Drive
Champaign, IL 61820

Dr. Grant Henning
Senior Research Scientist
Division of Measurement
Research and Services
Educational Testing Service
Princeton, NJ 08541

Ms. Rebecca Hetter
Navy Personnel R&D Center
Code 62
San Diego, CA 92152-6800

Dr. Paul W. Holland
Educational Testing Service
Rosedale Road
Princeton, NJ 08541

Prof. Lutz F. Hornke
Institut fur Psychologie
RWTH Aachen
Jaegerstrasse 17/19
D-5100 Aachen
WEST GERMANY

Dr. Paul Horst
677 G Street, #184
Chula Vista, CA 90010

Mr. Dick Hoshaw
OP-135
Arlington Annex
Room 2834
Washington, DC 20350

Dr. Lloyd Humphreys
University of Illinois
Department of Psychology
603 East Daniel Street
Champaign, IL 61820

Dr. Steven Hunka
Department of Education
University of Alberta
Edmonton, Alberta
CANADA

Dr. Huynh Huynh
College of Education
Univ. of South Carolina
Columbia, SC 29208

Dr. Robert Jannarone
Department of Psychology
University of South Carolina
Columbia, SC 29208

Dr. Dennis E. Jennings
Department of Statistics
University of Illinois
1409 West Green Street
Urbana, IL 61801

Dr. Douglas H. Jones
Thatcher Jones Associates
P.O. Box 6640
10 Trafalgar Court
Lawrenceville, NJ 08648

Dr. Milton S. Katz
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Prof. John A. Keats
Department of Psychology
University of Newcastle
N.S.W. 2308
AUSTRALIA

Dr. G. Gage Kingsbury
Portland Public Schools
Research and Evaluation Department
501 North Dixon Street
P. O. Box 3107
Portland, OR 97209-3107

Dr. William Koch
University of Texas-Austin
Measurement and Evaluation
Center
Austin, TX 78703

Dr. James Kraatz
Computer-based Education
Research Laboratory
University of Illinois
Urbana, IL 61801

Dr. Leonard Kroeker
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Daryll Lang
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Jerry Lehnus
Defense Manpower Data Center
Suite 400
1600 Wilson Blvd
Rosslyn, VA 22209

Dr. Thomas Leonard
University of Wisconsin
Department of Statistics
1210 West Dayton Street
Madison, WI 53705

END
DATE
FILMED
7-88
DTIC